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Atmospheric Limitations on Antenna Pointing Performance

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Atmospheric limitations to the pointing performance of a ground-based satellite antenna are analyzed in terms of power loss relative to perfect pointing. The antenna under consideration employs an open-loop tracking technique for pointing to a satellite with known position. A model is set up for the analysis to account for the statistical effect of atmospheric refraction of waves on the pointing performance of the ground antenna. An analytic result is obtained for prediction of the pointing performance. Measured atmospheric refraction data are incorporated into the model to demonstrate the atmospheric limitations to the highest antenna gain permissible within a specified pointing loss.

Introduction

THE gain of a reflector antenna is normally related directly to its aperture size and, in principle, can be increased without limit. Practical antennas, however, are limited by many factors. These include the limitations due to mechanical difficulties, such as the well-known root-mean-square (rms) error on the reflector surface, and the surface spatial correlation.[1] In contrast to the traditional method of analyzing antenna limitations from the viewpoint of physical realizability, this paper discusses a new factor which enters or arises as electromagnetic waves propagate from a transmit antenna to an intended receive aperture.

The study considers the limitations on usable gain of a ground-based narrow-beam (high-gain) antenna that employs an open-loop tracking algorithm to point to a distant, relatively small receiver aperture, such as a moderate gain antenna aboard a spacecraft. It is assumed that the open-loop tracking algorithm can calculate the exact satellite location. Perfect pointing can be achieved if the wave propagation in the atmosphere is ideal or if complete knowledge of the atmospheric parameters along the wave path is available. These conditions, however, are not typical in the operating environment. In practice, techniques that would not achieve perfect pointing are resorted to.

The ionosphere and the troposphere are generally known to affect the propagation of radio waves. Wave propagation experiences an angular path deviation (refractive bending) due to spatial inhomogeneities in the atmosphere. These inhomogeneities are continuously varying in a random manner as a function of time. In spite of the theoretical models and measured data that are available for estimation of refractive bending, there is still a significant amount of uncertainty regarding the exact path of waves propagating in the atmosphere. This uncertainty is generally expressed in statistical units as standard deviations or rms errors for a mean wave path in measured bending data or predictive models.[2-4] The inhomogeneous and fluctuating atmosphere that causes uncertainty in determining the exact wave path has been considered the ultimate limitation on the resolution in many radar applications.[5,6] As will be shown in this paper, the uncertainty also imposes a severe limitation on the usable gain of a narrow-beam ground-based satellite antenna.

This paper proposes a model for quantifying the pointing performance of a ground-based satellite antenna. Measured refractive bending data are incorporated into the model to present an example of how to determine the highest antenna gain permissible within the limit of a specified pointing loss.

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Model and Analysis

For ease of calculations, the model employed for this study assumes a rectangular cross-section beam, as shown in Fig. 1. This simulates the region between half-power (3 dB) points of a generalized antenna pattern. A normalized beam intensity, which varies linearly from 1 on axis to 1/2 at the beam edges, is employed to approximate the main beam intensity within 3 dB points. The approximation is reasonable with its mathematical simplicity and the fact that any discrepancy relative to practical distributions within 3 dB points is upper-bounded by 0.1.

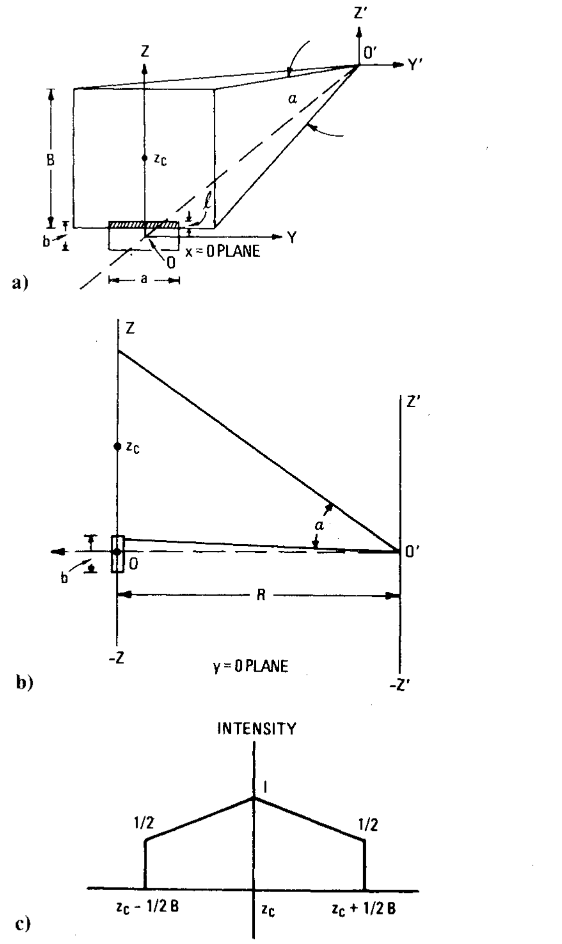

Fig. 1 Illustration of antenna beam and receiver aperature: a) perspective view, b) side view, and c) beam intensity distribution.

In the past, the propagation of waves from the ground antenna to the receive aperture has been considered ideally, i.e., the peak main beam intensity Z_c of the ground antenna would arrive exactly at the counterpoint of the receive aperture 0. Again, for mathematical tractability, a rectangular receiving aperture, such as a horn, is assumed for the satellite antenna. In this analysis, the fact Z_c would fluctuate, from an ensemble viewpoint, in elevation around 0 in a real atmospheric environment is taken into consideration and is characterized as a Gaussian random variable. The variable has a zero mean and a standard deviation, σ_ϵ, where σ_ϵ is approximately equal to the distance between the transmit and the receive antenna R, multiplied by the deviation of the apparent elevation angle σ_ϵ'. The deviation is due largely to random perturbations which occur during the course of wave propagation.

A performance index $\bar{r}$ is employed in the analysis to evaluate the performance of the ground antenna in a real atmosphere. The index is defined as the ratio of the average power received by the space antenna to the power that would be received under ideal propagation.

It is shown in the Appendix that the performance index $\bar{r}$ has the following form:

$$\bar{r}=2\int_0^{\frac{1}{2}\phi}\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}z^2}\mathrm{d}z-\frac{2}{\sqrt{2\pi}\phi}\left(1-e^{-\phi^2/8}\right) \tag{1}$$

where $\phi=\alpha/\sigma_\epsilon'$ is the ratio of the antenna beam width α to the standard deviation or rms of elevation-angle errors, σ_ϵ'.

An important assumption made in deriving Eq. (1) is that the height of the transmitted beam at the plane of the receive aperture is much greater than the receive aperture. This condition is generally met by antennas with low-to-moderate gains aboard a spacecraft at an altitude of at least 2000 km.

As shown in Eq. (1), $\bar{r}$ is a function of ϕ. Equation (1) indicates that the performance of an antenna operating in a real atmosphere will be subject to a constraint imposed by ϕ. More precisely, the performance index $\bar{r}$, as depicted in Fig. 2, reveals the following interesting results:

1) Antennas achieve near-optimum performance if $\phi\geq 8$. Ground-based antennas operating in this region pay minimal penalty to wave path uncertainty; the loss is bounded by 0.45 dB at $\phi\geq 8$ (10% below the ideal received power). Antennas operated in this region perform almost as well as those operated under ideal conditions. The traditional method of considering antenna pointing can be accurately applied.

2) Antennas have significantly degraded performance if $3\leq\phi<8$. The loss is 0.52 dB at $\phi=7$ and 1.64 dB at $\phi=3$.

3) Prohibitive antenna loss is involved if $1\leq\phi<3$. The loss incurred is from 2.8 dB at $\phi=2$ to 5.4 dB at $\phi=1$.

4) The antenna is likely to be useless for $\phi<1$, since it is apt to be pointed off the receiver. It is noted that for $\phi\ll 1$, $\bar{r}\cong 0.3\ \phi$. Since directivity goes up as ϕ^{-2}, there is still a net increase in gain with increasing antenna size (about 3 dB for every doubling of the antenna size for $\phi<3$). Thus, the limitation is set by cost and inability to point at the receiving antenna.

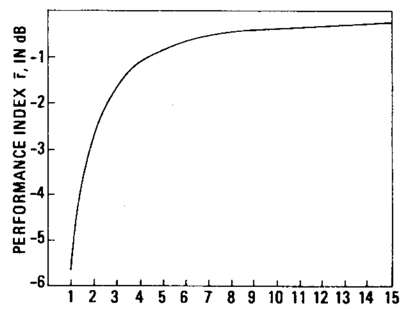

Fig. 2 Performance index $\bar{r}$ vs ϕ.

It should be noted that the preceding results were derived by considering the ground-based antenna in a transmit mode. The approach permitted easy visualization of the analysis. Since, in most cases, the reciprocity principle holds true for radio antennas, the preceding results are also applicable to the pointing efficiency of the ground-based antenna in a receive mode and the space antenna in the transmit mode.

Example of Application

The derived analytic result can be used to determine how high the gain of a practical antenna can be without suffering an intolerable loss in a real atmosphere. The ground-based antenna considered in the following example employs an open-loop tracking technique to point to a distant receiver. The receiver has a low-to-moderate gain antenna aboard a spacecraft whose altitude is at least 2000 km.

To facilitate discussion of the example, discussion of the relationship between the antenna gain and the 3-dB angular beam width is necessary.

At wavelength λ, the antenna gain G of a uniformly illuminated circular aperture is related to its diameter D by the following expression[7]:

$$G=\eta_A\eta_\epsilon(\pi D/\lambda)^2 \tag{2}$$

where η_A is the antenna aperture efficiency including feed taper, spillover, and total aperture blockage. η_ϵ is the antenna surface error factor. D/λ can be further related to the 3-dB angular beam width α, according to the following expression:

$$D/\lambda=58/\alpha \qquad \alpha\text{ in degrees} \tag{3}$$

Upon substituting Eq. (3) into Eq. (2), one obtains G in terms of α. Namely,

$$G=\eta_A\eta_\epsilon(182.2/\alpha)^2 \tag{4}$$

where α is in degrees. With $\phi=\alpha/\sigma_\epsilon'$ or $\alpha=\phi\sigma_\epsilon'$, Eq. (4) can be converted to

$$G=\eta_A\eta_\epsilon(182.2/\phi\sigma_\epsilon')^2 \tag{5}$$

where σ_ϵ' is also in degrees.

The usable antenna gain G_u can be obtained through incorporation of pointing loss $\bar{r}$ into G. Namely,

$$G_u=\bar{r}G\ =\bar{r}\eta_A\eta_\epsilon(182.2/\phi\sigma_\epsilon')^2 \tag{6}$$

Typical aperture efficiencies (η_A) of millimeter-wave, radome-enclosed antennas are of the order of 55-80%. The

Table 1 Refractive bending measured at Prospect Hill, Mass., deg[a]

Initial elevation angle	Mean bending	Standard deviation in mean model	rms error in linear model
1	0.486	0.056	0.039
5	0.185	0.014	0.008
10	0.099	0.007	0.003
20	0.047	0.003	0.001

[a] From L. Telford, personal communication, 1977.

Table 2 Maximum antenna gains vs elevation angle, $\phi=8$[a]

Initial elevation angle, deg	Bending estimation with mean model, dB		Bending estimation with linear model and surface refractivity, dB	
	G	G_u	G	G_u
1	49.7	49.2	52.8	52.3
5	61.7	61.2	66.6	66.1
10	67.7	67.2	75.1	74.6
20	75.1	74.6	84.6	84.1

[a] 70% aperture efficiency and 80% antenna surface error factor is assumed.

surface error factor (η_ϵ) degrades for a given antenna as operating frequencies increase. In the following example, 70% is assumed for η_A and 80% for η_ϵ. The assumed η_ϵ value is feasible at 100 GHz.[8]

As mentioned earlier, ϕ depends on the antenna beam width and also on the standard deviation or the rms error in estimating the refractive bending. For a given propagation path, the amount of standard deviation or rms error depends on the technique employed to predict the bending. Two techniques are commonly employed.

One technique is to employ known radio refractivity profiles or ray bending data for estimation without use of real-time environmental data. The profile or the data base is called a mean model in this paper because estimation is based mainly on the average value of a large set of data for an elevation angle of interest.

The other technique is to employ the real-time information on surface refractivity which is incorporated into a linear model for bending estimation. This technique exploits the fact that a significant part of the total refractive bending occurs near the surface. Statistical analysis of measured data has shown that the total bending is almost linearly correlated to the surface refractivity for a given apparent elevation angle.[2,3] The linear model therefore is a collection of such data which are reduced to show the characteristic of linear relationship between bending and surface refractivity. The resultant rms error or standard deviation from the linear model is smaller than that of the mean model because of the real-time environmental data.

Recent refraction data as shown in Table 1 are used in an application example to determine the highest permissible antenna gain within a specified pointing loss. Telford took the measurements by sun tracking at the Prospect Hill Observatory of the Air Force Rome Air Development Center, Waltham, Mass. The table shows the average bending, standard deviation in the mean model, and rms error in the linear model as a function of initial elevation angles for the 100 sets of data. The data were taken mainly at 15 GHz, but they will generally hold for operating frequencies in the microwave region (over 2 GHz), which is of central concern in this example. Note that the total refractive bending, as reported in Table 1, is equal to elevation-angle error, because pointing is made to a space source.

Since the rms error becomes smaller as the initial elevation angle gets higher, it is sufficient for this example to examine initial elevation angles up to 20 deg. Radio refractivity profiles vary from one location to another, and the results derived in this example should not be interpreted as representative of any other locale of interest.

Maximum antenna gain (G) and maximum usable gain (G_u) which are permissible within a specified pointing loss are calculated for each initial elevation angle, as shown in Table 2, for $\phi = 8$, near-optimum pointing performance, and in Table 3 for $\phi = 3$, 1.64 dB pointing loss performance. It is seen that for near-optimum pointing performance ($\phi \geq 8$), an antenna with 67.7 dB gain can operate at 10 deg and higher initial elevation angles if known refractivity profiles are employed. On the other hand, if surface refractivity measurements are incorporated, the maximum gain can be increased to 75.1 dB under similar operating conditions. With similar operational conditions, but for a more degraded performance ($\phi \geq 3$), the 67.7 dB antenna can be increased to 76.3 dB and the 75.1 dB antenna to 83.6 dB. Higher gains still can be achieved, but a much less usable gain than would be expected from antenna design considerations alone could be realized because of high pointing loss as shown in Table 4 for $\phi = 1$.

Table 3 Maximum antenna gains vs elevation angle, $\phi = 3$[a]

Initial elevation angle, deg	Bending estimation with mean model, dB		Bending estimation with linear model and surface refractivity, dB	
	G	G_u	G	G_u
1	58.2	56.5	61.3	59.6
5	70.2	68.5	75.1	73.4
10	76.3	74.6	83.6	81.9
20	83.6	81.9	93.2	91.5

[a] 70% aperture efficiency and 80% antenna surface error factor is assumed.

Table 4 Maximum antenna gains vs elevation angle, $\phi = 1$[a]

Initial elevation angle, deg	Bending estimation with mean model, dB		Bending estimation with linear model and surface refractivity, dB	
	G	G_u	G	G_u
1	67.7	62.3	70.9	65.5
5	79.8	74.4	84.6	79.2
10	85.8	80.4	93.2	87.7
20	93.2	87.8	102.7	97.3

[a] 70% aperture efficiency and 80% antenna surface error factor is assumed.

Conclusions

A statistical model has been set up for analyzing the pointing performance of a large ground-based satellite antenna operated in a real atmosphere. The antenna considered employs an open-loop tracking algorithm. The results are directly applicable to determining how high the gain of a practical antenna can be without suffering an intolerable pointing loss.

The maximum antenna gain permissible within a specified pointing loss is highly dependent on the initial elevation angle as well as path prediction techniques. Based on the data at one site for initial elevation angles of 10 deg or higher, the maximum antenna gain with near-optimum pointing performance is 67.7 dB if known radio refractivity profiles are employed for path prediction and 75.1 dB if surface refractivity measurements are also incorporated. Under similar operating conditions, but allowing for more degraded performance, the antenna gain can be increased to 76.3 dB and even to 83.1 dB with measured surface refractivity data.

Recent advanced antenna construction techniques allow antenna gains to achieve 78-82 dB.[8,9] One can conclude from this initial study that there is a need to bring propagation factors into the design consideration of future narrow-beam ground-based antennas for space applications, particularly those employing open-loop tracking techniques and operating at low elevation angles.

Appendix: Derivation of Performance Index

This Appendix is provided to indicate the procedures used in deriving the performance index $\bar{r}$. This index is the ratio of the average-received power in real atmospheric conditions, $\overline{P(\alpha,\sigma_\epsilon')}$, to the power received in an ideal situation assuming no path errors, $P(\alpha,0)$. Before actually deriving the expression for $\bar{r}$, some preparatory calculations will be presented.

Formulation of Beam Intensity Function

The beam intensity is described by k_1 and k_2, as was shown in Fig. 1c, where

$$k_1(z,z_c) = \frac{1}{B}(z+B-z_c) \qquad \left(z_c - \frac{B}{2}\right) \leq z \leq z_c$$

$$k_2(z,z_c) = \frac{1}{B}(-z+B+z_c) \qquad z_c \leq z \leq \left(z_c + \frac{B}{2}\right) \qquad \text{(A1)}$$

Formulation of Height of Illumination

By inspecting Fig. 1c, the height of illumination by beam on receiving antenna, denoted by $l(z_c)$, is found to be

$$l(z_c) = b \qquad \text{if } 0 < |z_c| \leq \tfrac{1}{2}(B-b) \qquad \text{(A2a)}$$

$$l(z_c) = \tfrac{1}{2}\,(B+b) - z_c \quad \text{if } \tfrac{1}{2}\,(B-b) \le |z_c| \le \tfrac{1}{2}\,(B+b) \qquad \text{(A2b)}$$

$$l(z_c) = 0 \qquad \text{otherwise} \qquad \text{(A2c)}$$

Figure A1 depicts conditions (A2a-A2c).

Derivation of $P(\alpha,0)$

The beam intensity function is assumed to be symmetrical with respect to the Y-axis, as shown in Fig. A2. The intensity of the beam illumination on the receive aperture along the Y axis is assumed to be constant across the width of the receive aperture a. This assumption is practical for this analysis because the analysis is not concerned with the relatively small azimuthal pointing errors. Also, the size of the beam illumination on the receive aperture is considered to be much larger than the receive aperture. The derivation of $P(\alpha,0)$ follows:

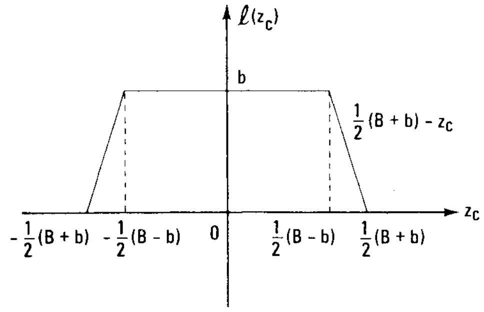

Fig. A1 Plot of $l(z_c)$.

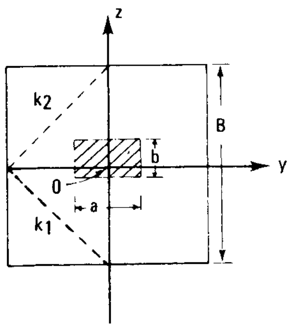

Fig. A2 Symmetry of beam intensity with respect to y axis.

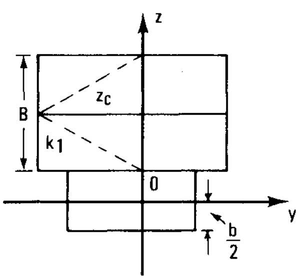

Fig. A3 $z_c = 1/2(B+b)$.

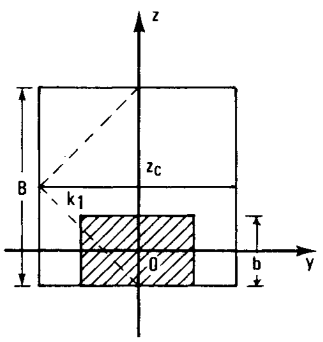

Fig. A4 $z_c = 1/2(B-b)$.

$$P_r(\alpha,0) = 2\int_0^{b/2} a\cdot k_2(z,0)\,\mathrm{d}z = 2a\int_0^{b/2}\left(1-\frac{z}{B}\right)\mathrm{d}z$$

$$\cong ab \qquad \text{assuming } B \gg z \text{ for } 0 \le z \le b/2 \qquad \text{(A3)}$$

Note that in Fig. A2 and other figures following, dotted lines represent regions defined by k_1 or k_2.

Derivation of $\overline{P(\alpha,\sigma_\epsilon')}$

As z_c is a random variable ranging theoretically from $-\infty$ to $+\infty$ on the z axis, the height of beam illumination on the receiving aperture at any value of z_c is described in Eq. (A2) by $l(z_c)$. It is seen in Fig. A1 that $l(z_c) = 0$ if $|z_c| \ge \tfrac{1}{2}(B+b)$. Therefore, for calculation of $P(\alpha,\sigma_\epsilon')$, only $0 \le |z_c| \le \tfrac{1}{2}(B-b)$ must be considered. (Bars in this paper denote the average value.) Because of the discontinuity of intensity function at z_c, the following breakdown of regions is necessary.

1) For $\tfrac{1}{2}(B-b) \le z_c \le \tfrac{1}{2}(B+b)$, see Figs. A3 and A4. Let $P_{r1}(z_c)$ denote the power received at values of z_c just defined, and assume $B/2 > b$; it follows that

$$P_{r1}(z_c) = a\int_{(b/2)-l(z_c)}^{b/2} k_1(z,z_c)\,\mathrm{d}z$$

$$= \frac{a}{B}\int_{z_c-(B/2)}^{b/2} (z+B-z_c)\,\mathrm{d}z$$

$$= \frac{a}{B}\left[\frac{z_c^2}{2} - z_c\left(B+\frac{b}{2}\right) + \frac{3}{8}B^2 + \frac{b^2}{8} + \frac{Bb}{2}\right]$$

$$\overline{P_{r1}(z_c)} = \int_{(B-b)/2}^{(B+b)/2} P_{r1}(z_c)\frac{1}{\sqrt{2\pi}}\exp\left[-\frac{1}{2}\left(\frac{z_c}{\sigma_\epsilon}\right)^2\right]\mathrm{d}z_c$$

$$P_{r1}(z_c) \text{ is negligible if } B \gg b \qquad \text{(A4)}$$

2) For $b/2 \le z_c \le \tfrac{1}{2}(B-b)$, see Fig. A5. Letting $P_{r2}(z_c)$ denote the power received at values of z_c just defined, and assuming $B/2 > b$, it follows that

$$P_{r2}(z_c) = a\int_{-b/2}^{b/2} k_1(z,z_c)\,\mathrm{d}z = ab\left(1-\frac{z_c}{B}\right) \qquad \text{(A5)}$$

3) For $0 \le z_c \le b/2$, see Fig. A6. Letting $P_{r3}(z_c)$ be the power received at values of z_c just specified, it follows that

$$P_{r3}(z_c) = a\int_{-b/2}^{z_c} k_1(z,z_c)\,\mathrm{d}z + a\int_{z_c}^{b/2} k_2(z,z_c)\,\mathrm{d}z$$

$$= ab - \frac{ab^2}{4B} - \frac{az_c^2}{B} \approx ab \quad \text{if} \quad B \gg a \text{ and } b \qquad \text{(A6)}$$

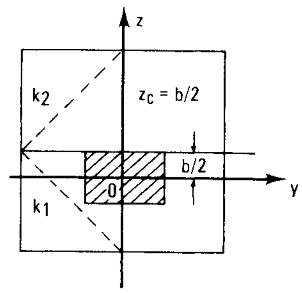

Fig. A5 $z_c = b/2$.

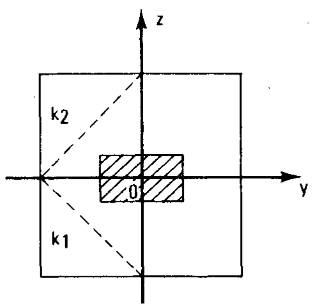

Fig. A6 $z_c = 0$.

Having derived P_{r1}, P_{r2}, and P_{r3}, one can now start to formulate $\overline{P_r(\alpha,\sigma_\epsilon')}$. Taking advantage of the symmetrical properties this problem has, one can determine that

$$\overline{P_r(\alpha,\sigma_\epsilon')} = 2[\overline{P_{r1}(z_c)} + \overline{P_{r2}(z_c)} + \overline{P_{r3}(z_c)}] \qquad \text{(A7)}$$

Since $\overline{P_{r1}(z_c)} \approx 0$, as shown in Eq. (A4), Eq. (A7) becomes

$$\overline{P_r(\alpha,\sigma_\epsilon')} = 2[\overline{P_{r2}(z_c)} + \overline{P_{r3}(z_c)}]$$

$$= 2\int_{b/2}^{(B-b)/2} \left(ab - \frac{abz_c}{B}\right) \frac{1}{\sqrt{2\pi}\sigma_\epsilon} \exp[-\tfrac{1}{2}(z_c/\sigma_\epsilon)^2]dz_c$$

$$+ 2\int_0^{b/2} ab \frac{1}{\sqrt{2\pi}\sigma_\epsilon} \exp[-\tfrac{1}{2}(z_c/\sigma_\epsilon)^2]dz_c$$

$$= 2ab\int_0^{(B-b)/2} \frac{1}{\sqrt{2\pi}\sigma_\epsilon} \exp[-\tfrac{1}{2}(z_c/\sigma_\epsilon)^2]dz_c$$

$$- 2\frac{ab}{B}\int_{b/2}^{(B-b)/2} \frac{z_c}{\sqrt{2\pi}\sigma_\epsilon} \exp[-\tfrac{1}{2}(z_c/\sigma_\epsilon)^2]dz_c \qquad \text{(A8)}$$

Letting $z_c/\sigma_\epsilon = z$ and assuming $B \gg b$ and $\sigma_\epsilon \gg b$, Eq. (A8) becomes

$$\overline{P_r(\alpha,\sigma_\epsilon')} \cong 2ab\int_0^{\frac{1}{2}\alpha/\sigma_\epsilon'} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{z^2}{2}\right)dz$$

$$- 2ab\frac{\sigma_\epsilon'}{\alpha}\int_0^{\frac{1}{2}\alpha/\sigma_\epsilon'} \frac{z}{\sqrt{2\pi}} \exp\left(-\frac{z^2}{2}\right)dz$$

$$= 2ab\left[\int_0^{\phi/2} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{z^2}{2}\right)dz - \frac{1}{\sqrt{2\pi}\phi}\left(1-e^{-\phi^2/8}\right)\right] \qquad \text{(A9)}$$

Note, in the derivation of Eqs. (A8) and (A9), it has been considered that

$$\sigma_\epsilon \cong R\cdot\sigma_\epsilon', \quad B \cong R\cdot\alpha, \quad \text{and} \quad \phi = \alpha/\sigma_\epsilon' \qquad \text{(A10)}$$

where R is the distance as defined in Fig. 1.

Derivation of $\bar{r}$

$\bar{r}$ is defined in this paper as

$$\bar{r} = \overline{P(\alpha,\sigma_\epsilon')}/P(\alpha,0) \qquad \text{(A11)}$$

Using the results of Eq. (A3) and Eq. (A9), one can find that

$$\bar{r} = 2\left[\int_0^{\phi/2} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{z^2}{2}\right)dz - \frac{1}{\sqrt{2\pi}\phi}\left(1-e^{-\phi^2/8}\right)\right] \qquad \text{(A12)}$$

This completes the derivation of $\bar{r}$.

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References

[1]Ruze, J., "Antenna Tolerance Theory—A Review," *Proceedings of the IEEE,* Vol. 54, April 1966, pp. 663-640.

[2]Bean, B.R. and Dutton, E.J., *Radio Meterology,* NBS Monograph 92, March 1966, pp. 49-88.

[3]Crane, R.K., "Refraction Effects in the Neutral Atmosphere," *Methods of Experimental Physics,* Vol. 12, Part B, edited by M.L. Meeks, Academic Press, New York, 1976, pp. 186-200.

[4]Gallop, M.A. and Telford, L.E., "Use of Atmospheric Emission to Estimate Refractive Errors in a Non-Horizontally Stratified Troposphere," *Radio Science, 10,* Nov. 1975, pp. 935-945.

[5]Ruina, J.P. and Angulo, C.M., "Antenna Resolution as Limited by Atmospheric Turbulence," *IEEE Transactions on Antennas and Propagation,* March 1963, pp. 153-161.

[6]Wyman, P.W., "Analysis of the Problem of Pointing a Laser Radar," U.S. Naval Research Laboratory, Washington, D.C., DDC MD40076, Oct. 7, 1965.

[7]Kraus, J.D., *Antennas,* McGraw-Hill, New York, 1950, pp. 343-346.

[8]Hugeunin, G.R. and Cohen, A., "The Millimeter-Wave Telescope at the Five College Radio Astronomy Observatory," *Proceedings of URSI,* Amherst, Mass., Oct. 1976, p. 112.

[9]Menzel, D.H., "A New Radio Telescope for Sweden," *Sky and Telescope,* Oct. 1976, pp. 240-242.